May 8, 1856.

The LORD WROTTESLEY, President, in the Chair.

In accordance with the Statutes, the Secretary read the following list of Candidates recommended by the Council for election into the Society:—

John Hutton Balfour, M.D.
Edward W. Binney, Esq.
Sir John Bowring.
Sir John Fox Burgoyne, Bart.
Philip Henry Gosse, Esq.
Robert Harkness, Esq.
Cæsar Henry Hawkins, Esq.
Manuel John Johnson, Esq.
John Carrick Moore, Esq.
Henry Minchin Noad, Esq.
Edmund Potter, Esq.
Rev. T. Romney Robinson, D.D.
Henry Hyde Salter, M.D.
Archibald Smith, Esq.
Capt. Thomas A. B. Spratt, R.N.

The following communications were read:—

I. "On various Phenomena of Refraction through Semi-lenses or Prisms, producing anomalies in the illusion of Stereoscopic Images." By A. CLAUDET, Esq., F.R.S. Received April 22, 1856.

(Abstract.)

The author having observed that photographic pictures representing flat surfaces, when examined in the refracting stereoscope, have the appearance of concavity, has endeavoured to discover the cause of that phenomenon, and to explain it.

In order to ascertain if this peculiar effect was attributable to some imperfection in the lenses of the camera obscura which had produced the photographic pictures, or to a property of the stereoscope itself, he began to test the stereoscope without photographic

images. For this experiment he placed under each tube of the stereoscope a diagram composed of vertical and horizontal lines crossing each other.

The two diagrams, perfectly identical when seen in the stereoscope, coalesced and formed only one figure; but although each diagram, when seen separately by its corresponding eye, appeared perfectly flat, still the coalescing image of the two presented a surface conspicuously concave; consequently there was no doubt that the same illusion observed in photographic pictures was due only to the effect of the stereoscope. This experiment was decisive, and it remained to discover how the illusion was produced. The investigations showed that the phenomenon, which is a defect detrimental to the beauty and correctness of the stereoscopic representations and unavoidable in the refracting stereoscope, is a plain illustration of the cause of relief and distance, and yield the clearest explanation of the stereoscopic illusion,—proving that it is founded on the true principles of natural binocular vision.

When we look through a prism placed near the eye at a straight line, the refracting edge of the prism being parallel with the straight line, that line is refracted laterally and appears bent, with its concave side turned to the thin edge of the prism. The two tubes of the stereoscope being supplied with semi-lenses acting as prisms, each lens bends all vertical straight lines, and the concave sides of these lines are turned towards the thin edges of the lenses, and consequently towards each other. When we examine in the stereoscope two curved lines having their concave sides turned towards each other, the result of the coalescing of these two lines is a concave line, the extremities appearing nearer and the centre further. If the convex sides are turned towards each other, the result of the coalescence is a convex line, the extremities appearing further and the centre nearer. By the same reason, if straight lines are bent by the prismatic refraction of the two semi-lenses, as the bending is effected so that the concave sides are turned towards each other, the result is by coalescence a concave line. The two photographic images will have all their vertical lines bent in the same manner, and the stereoscope will give the illusion of a picture represented on a concave surface.

When we look at natural objects, the optical axes have to con-

verge more for the nearest than for the furthest, in order to obtain a single vision by bringing the same object on the centre of each retina; therefore by habit we judge of the distances by the angle formed by the optical axes required to obtain a single vision. Again, while we look at one object, while other objects in the same line are situated before and behind that object, we have the sensation of their double images on the two retinæ. The double images of nearer objects are situated in the following order: one on the right of the centre of the right retina, and the other on the left of the centre of the left retina; and the double images of further objects, one on the left of the right retina, and the other on the right of the left retina.

In looking at the two pictures in the stereoscope, we have to converge the optical axes on one point which is beyond the plane of the pictures, so that two of their correspondent or similar points appear respectively on the two lines forming the angle of convergence of the optical axes, and each of these points is represented on the centre of one retina. As the two corresponding points of the two pictures are laterally nearer each other for the first plane and more distant for the receding plane, it follows that the optical axes have to converge beyond the plane of the pictures on a nearer point for the first, and on a further point for the last. Therefore, the angle of convergence by which similar points of the two pictures appear on each axis and consequently fall on the centre of each retina, conveys the sensation of their respective distances; more convergence indicates less distance, and less convergence more distance. All the other corresponding points of the two pictures which are not on the optical axes or on similar points of the two retinæ, form double images; and when we look at one point, all the points of nearer and further planes appear double in the same order on the two retinæ, as when we look in like manner at natural objects; and the situation of double images seen through the stereoscope indicates the distances of the objects they represent, according as one is on the right of the right retina and the other on the left of the left retina, or one on the left of the right retina and the other on the right of the left retina.

This being explained, it is easy to understand what will be the stereoscopic result of vertical lines represented as curved, and having their concavities turned toward each other. The two correspondent points of the top and bottom of the two concave lines, being nearer

each other, will require more convergence than the two correspondent points of the centres of the concave lines, and will appear nearer, whilst the two points of the centre requiring less convergence will appear further; the intermediate points from the centres to the extremities of the two bent lines will appear gradually less distant, therefore the coalescence of the two lines bent laterally will produce the illusion of a single line conspicuously concave, in a vertical plane at right angles with the plane of the two separate lines.

Having demonstrated that the semi-lenses of the stereoscope, like prisms, bend laterally all the vertical lines of which the photograph pictures are composed, and that these lines in the two pictures present their concavity to each other, it is evident that the coalescence of the two images must give the illusion of a concave image.

The phenomenon of the lateral curvature given to vertical lines by the refraction of a prism, which vertical lines, when examined with two prisms, one for each eye, appear by coalescence as one line concave in a vertical plane at right angles with the plane of the two separate bent lines, can be curiously illustrated by the following experiment:—

If, holding in each hand one prism, the two prisms having their thin edges towards each other, we look at the window from the opposite end of the room, we see first two windows with their vertical lines bent in contrary directions; but by inclining gradually the optical axes, we can converge them until the two images coalesce, and we see only one window; as soon as they coincide, the lateral curvature of the vertical lines ceases, and they are bent projectively from back to front: we have then the illusion of a window concave towards the room, such as it would appear reflected by a concave mirror.

There is another phenomenon which can be noticed when looking at photographic pictures in the stereoscope; sometimes the picture appears to project out and sometimes to recede from its mountings. The first effect lessens the illusion, and the second renders it more effectual; therefore it is desirable to inquire how we can avoid the one and ensure the other.

We know that the distance of objects is in an inverse ratio with the angle of convergence required to see them single; also that with symmetrical figures or photographic pictures, when the horizontal or lateral distances of the several corresponding points is different, the

points less separated will appear nearer, and the more separated will appear further.

Suppose the two correspondent vertical lines of the openings or frames of the pictures be more distant than the two correspondent points of the furthest plane of the pictures themselves, then the openings or frames will appear behind the pictures; and suppose the correspondent vertical lines of the openings be less distant than the two correspondent points of the nearest plane of the pictures themselves, then the openings or frames will appear before the picture.

Therefore, when we wish to have the picture appearing behind the openings or their mountings, we have only to take care that the correspondent vertical lines of the mountings should be laterally less distant than the two correspondent points of the first plane of the picture. This can be easily done by taking the measure of the two correspondent points of the first plane by means of a pair of compasses, and tracing the two pairs of correspondent vertical lines bounding the openings, after having slightly reduced the angle of the compasses.

A very simple experiment may show the cause of the illusion of concavity of flat surfaces when examined through semi-lenses, and further prove that semi-lenses may give alternately the illusion of concavity and convexity according to the position of their thin edges; of concavity when their edges are towards each other, and of convexity when they are placed contrariwise. For this experiment we have only to employ a pair of those spectacles mounted with a spring whereby they are held on the nose.

When we read, holding such spectacles with both hands, we may by the elasticity of the spring adjust the two lenses so that the pupils of the eyes can coincide, first, with the two nearest edges, secondly, with the two centres, and thirdly, with the two furthest edges of the lenses.

In the first case, the page of the book will appear concave, because the pupils will look through the thin edges of the lenses which bend the vertical lines with their concave sides turned towards each other; in the second, the page will appear flat, because the pupils will look through the centres of the lenses which show the vertical lines perfectly straight; and in the third case, the page of the book will appear convex, because the pupils will look through the thin edges of

the lenses which bend the vertical lines with their convex sides turned towards each other.

These considerations have led the author to construct a stereoscope which presents flat surfaces perfectly flat. This new stereoscope has two entire lenses instead of two semi-lenses, and the eyes look through the centre of such lenses. The images not being laterally refracted, as in the semi-lenticular stereoscope, their coalescence requires a certain effort of divergence, or to squinting outwards, which a little practice will enable us to perform easily. Persons capable of using this kind of stereoscope will see a picture whose surface is perfectly flat with all the illusion of relief and distance.

All lenses being more or less subject to the defect of bending straight lines when refracted by all the various points of their surface but the centre, and in a greater degree as those points are nearer the edges, it results that when images are produced in the camera obscura by the various points of the whole aperture, they will be bent in various contrary directions, and a certain confusion must arise injurious to the delicacy and correctness of the whole compound image. This may be proved by the following experiments:—If we take the image of a window by a small aperture placed on the right edge of a lens, say of 3 inches aperture, and another image of the same window, by placing the aperture on the left, taking care to shift the camera so that the two apertures will be exactly on the same line, we shall have two images of the same window apparently identical; but in placing these two images side by side in the central lens-stereoscope above described, first the image of the left side aperture on the right, and that of the right side aperture on the left, secondly the images *vice versâ*, we shall see in the first case a concave window, and in the second a convex window. But in examining the two images in the semi-lenticular stereoscope, we shall see in one case a concave window, and in the other a perfectly flat window, because in the first case the stereoscope will have increased the bending of the vertical lines of the two images, and in the second the stereoscope will have corrected the bending.

This fact naturally suggests the possibility of correcting the defect of the refracting stereoscope; for if the images of the camera were taken by semi-lenses, the bend resulting from this mode of operating might be corrected by the bend of the stereoscope, care being taken

to turn the thin edge of the semi-lenses of the two cameras in the direction which will produce a bending contrary to that of the semi-lenses of the stereoscope.

Having shown how the lateral proportional distances of any two correspondent points of the two stereoscopic pictures are the indices of their perspective distances, if we were, while looking in the stereoscope, to produce a change in those proportional lateral distances by sliding horizontally in a contrary direction, two pairs of superposed glass photographic pictures, the objects would appear to move, not in the horizontal lateral direction of that change which they naturally have, but in a straight line forward and backward, as if the object was approaching or receding.

But the most curious effect of that motion would be, that the objects would appear increasing in size while they were receding, and diminishing while approaching, which we know is contrary to the rule of perspective. This is another illusion entirely physiological, and the cause of which may be thus explained; while the object appears moving forward and backward it remains always the same size, but as we expect when it moves forward that it should increase in size, and when it moves backward that it should decrease, and as it does not, we feel that it is diminishing when approaching and increasing when receding.

II. "A Memoir upon Caustics." By Arthur Cayley, Esq., F.R.S. Received May 1, 1856.

(Abstract.)

The principal object of this memoir, which contains little or nothing that can be considered new in principle, is to collect together the principal results relating to caustics *in plano*, the reflecting or refracting curve being a right line or a circle, and to discuss with more care than appears to have been hitherto bestowed upon the subject, some of the more remarkable cases. The memoir contains in particular researches relating to the caustic by refraction of a circle for parallel rays, the caustic by reflexion of a circle for rays proceeding